

Mental Math I - Addition

1. Master all single digit combinations up to 10
 - $3 + 5 = 8$, $2 + 7 = 9$, etc.
2. Master number combinations to 10
 - $7 + 3$, $2 + 8$, etc.
 - $7 + ? = 10$
3. Break down numbers combinations over 10:
 - $8 + 3 = 8 + 2 + 1 = 10 + 1 = 11$
 - $6 + 7 = 3 + 3 + 7 = 3 + 1 = 13$
 - Learn doubles ($6 + 6$, $7 + 7$ etc.) , so $6 + 7 = 6 + 6 + 1$
4. Add tens
 - $13 + 10 = 3 + 10 + 10 = 3 + 20 = 23$
 - $10 + 8 + 7 = 10 + 8 + 2 + 5 = 10 + 10 + 5 = 25$
5. Rearrange to get 10's
 - $6 + 3 + 4 = 6 + 4 + 3 = 10 + 3 = 13$
 - $8 + 5 + 2 = 8 + 2 + 5 = 10 + 5 = 15$
 - $18 + 5 + 2 = 18 + 2 + 5 = 20 + 5 = 25$
 - $8 + 5 + 2 + 5 = 8 + 2 + 5 + 5 = 10 + 10 = 20$
6. Add nines, and then eights by subtracting the distance to 10
 - $13 + 9 = 13 + 10 - 1 = 23 - 1 = 22$
 - $24 + 8 = 24 + 10 - 2 = 34 - 2 = 32$
7. Break down a 1-digit plus a two-digit number
 - $17 + 8 = 17 + 3 + 5 = 20 + 5 = 25$
 - $5 + 18 = 3 + 2 + 18 = 3 + 20 = 23$
 - Or: $\quad = 5 + 8 + 10 = 13 + 10$
8. Add by subtracting the distance to 10 (start with 9's then 8's)
 - $5 + 19 = 5 + 20 - 1$
 - $17 + 8 = 17 + 10 - 2 = 27 - 2 = 25$
9. Break down two 2-digit numbers
 - $14 + 18 = 10 + 10 + 4 + 6 + 2 = 30 + 2 = 32$
Or: $\quad = 10 + 10 + 4 + 8 = 20 + 12 = 32$
Or: $\quad = 14 + 16 + 2 = 30 + 2 = 32$
Or: $\quad = 14 + 20 - 2 = 34 - 2 = 32$
 - $34 + 48 = 30 + 40 + 4 + 8 = 70 + 12 = 82$
Or: $\quad = 34 + 40 + 8 = 74 + 6 + 2 = 82$
Or: $\quad = 34 + 50 - 2 = 84 - 2 = 82$
10. Transfer units from one addend to another (“stealing”)
 - $8 + 14 = 10 + 12 = 22$
 - $19 + 12 = 20 + 11 = 31$
 - $99 + 22 = 100 + 21 = 121$

Mental Math II- Subtraction

1. Master all single digit differences up to 10
 - $8 - 5$, $7 - 2$, etc.
2. Master number differences to 10
 - $10 - 7$, $10 - 6$, etc.
3. Break down numbers combinations over 10:
 - $\underline{12 - 3} = 12 - 2 - 1 = 10 - 1 = 9$
 - $\underline{25 - 8} = 25 - 5 - 3 = 17$
4. Subtract tens
 - $\underline{23 - 10} = 13$
 - $\underline{37 - 20} = 37 - 10 - 10 = 17$
5. Subtract nines (minus 10, plus 1) and then eights (minus 10, plus 2)
 - $\underline{13 - 9} = 13 - 10 + 1 = 3 + 1 = 4$
 - $\underline{24 - 8} = 24 - 10 + 2 = 14 + 2 = 16$
- 6, Break down two 2-digit numbers
 - $\underline{34 - 18} = 34 - 10 - 8 = 24 - 8 = 16$
Or: $= 34 - 20 + 2 = 16$
Or: $= 34 - 4 - 4 - 10 = 16$
Or: $= 10 + 2 + 4 = 16$ (using a number line)
7. Add or subtract units from BOTH numbers
 - $\underline{29 - 14} = 30 - 15 = 15$ (add one to each)
 - $\underline{34 - 28} = 30 - 24 = 6$ (subtract 4 from each)
 - $\underline{103 - 26} = 100 - 23 = 100 - 20 - 3 = 77$

Mental Math III- Multiplication

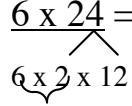
1. Number tricks

- Times 4 is times 2 twice.
- Times 6 is times 3, then times 2.
- Times 8 is times 2 three times.
- Times 5 is times 10, divide by 2.
- Times 3 is times 2, plus the number
- Times 9 is times 10, minus the number
- 11 times a 2-digit number: put first digit in front, second at end, add the two digits for the middle number in the answer
 - $23 \times 11 = 2_3 \rightarrow 253$

2. Single digit times a 2-digit number: Times the 10's, times the 1's:

- $\underline{6 \times 24} = 6 \times 20 + 6 \times 4 = 120 + 24 = 144$
- $\underline{12 \times 15} = 10 \times 15 + 2 \times 15 = 180$

3. Double and half:

- $\underline{6 \times 24} = 12 \times 12 = 144$ (you can always double one factor and halve the other)

- $\underline{4 \times 16} = 8 \times 8 = 64$
- $\underline{5 \times 24} = 10 \times 12$ (*very useful* - it's easy to mult by 5)

4. Break down 2-digit numbers into factors:

- $\underline{12 \times 24} = 12 \times 12 \times 2 = 144 \times 2 = 288$
- $\underline{6 \times 18} = 6 \times 9 \times 2 = 54 \times 2 = 108$
- $\underline{12 \times 25} = 12 \times 5 \times 5 = 60 \times 5 = 300$
- Or: $\quad = 3 \times 4 \times 25 = 3 \times 100 = 300$

Mental Math IV- Division

1. Number tricks

- Dividing by 4 : divide by 2 twice.
- Dividing by 8 : divide by 2 three times.

2. Briefly disregard zeroes at the end of a number

- $240 \div 3$ is $24 \div 3$, then put back the zero

3. Break up dividend before dividing

- $240 \div 3 =$
$$\frac{24 \times 10}{3} = \frac{8 \times 24 \times 10}{3}$$

In other words, break up the dividend into factors, and then only divide ONE of the factors by the divisor.

- Verify that this works with smaller numbers: $80 \div 2 = \frac{4 \times 8 \times 10}{2}$

To understand this: 80 is 8 x 10, Half of it is 4 x 10, not 4 x 5.

More examples:

- $320 \div 16 = \frac{16 \times 2 \times 10}{16} = 2 \times 10 = 20$
- $880 \div 22 = \frac{22 \times 4 \times 10}{22} = 40$

3. Learn short division

- $$\begin{array}{r} 214 \\ 2 \overline{)428} \end{array}$$

Say:

- $$\begin{array}{r} 269 \\ 2 \overline{)5138} \end{array}$$
 - “2 goes into 5 two times with one left over”
 - Write the “1” in front of the next digit
 - “2 goes into 13 six times with one left over”
 - Write the “1” in front of the next digit
 - “2 goes into 18 nine times with 0 left over. Done.

With practice, students can learn to do this in their heads.